

PROBLEM SETS 3. DUE FRIDAY 8 SEPTEMBER

PROBLEM SET 3. PROBLEMS FROM LECTURE 3.

Reading. *Quick Calculus*, pp. 97–129.

Supplementary reading. Simmons, Chapter 3.

1. Differentiate the following functions, using the rules we learned in lecture today.
 - (a) $y = 3x^4 + 2x^3 - x^2 + 4x - 7$
 - (b) $y = (2x^2 + 3)(3x^4 - 2x - 5)$
 - (c) $y = \frac{3x-7}{2x^2+4}$
 - (d) $y = (10x - 2)^5(3x^2 - 1)^2$
 - (e) $y = \sec \theta \csc \theta$
 - (f) $y = \tan \theta = \frac{\sin \theta}{\cos \theta}$
 - (g) $y = e^{5x+7}$
 - (h) $y = \ln\left(\frac{3x^2}{4x+2}\right)$
 - (i) $y^3 = \sqrt{2xy - 4xy^2}$
 - (j) $y = (x^2 + 4)^{\frac{5}{2}}$
2. Given a cubic equation $f(x) = ax^3 + bx^2 + cx + d$, for what constants a , b , c , and d does the graph of $f(x)$ have exactly
 - (a) two horizontal tangents?
 - (b) one horizontal tangent?
 - (c) no horizontal tangents?(Hint: A horizontal tangent to the graph occurs when the derivative $f'(x) = 0$.)
3. Find the values of x for which the graph $f(x) = x + 2 \sin(x)$ has a horizontal tangent.
4. Find the tangent line to

$$f(x) = \frac{x^3 + x}{x - 1}$$

at the point $(2, 10)$.

5. We have talked about the tangent line to a graph at some point P on the graph. The *normal line* to a graph at the point P is the line that is perpendicular to the tangent line to the graph at P . Given a line $f(x) = mx + b$, the perpendicular line $g(x)$ to $f(x)$ at P is the line with slope $-\frac{1}{m}$, also going through P . (See the figure on the next page.) Find the tangent line and the normal line to the graph $y = \frac{6}{x+2}$ at the point $(1, 2)$.

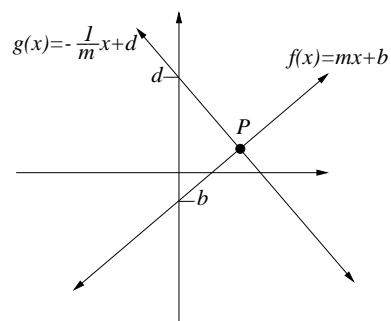


FIGURE 1. This shows the line $f(x) = mx + b$ and the perpendicular line $g(x) = -\frac{1}{m}x + d$, where d is determined by our choice of P .